Measurements of Ensemble Averaged Flame Dynamics using Spatially Resolved Analysis

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Abstract

When applying flame sheet models to predict the dynamics of turbulent flames, it is common to model turbulence using ensemble averaging of the velocity. Measurements of the flame dynamics were made to support use this type of methodology, by measuring the dynamic volume of the flame using phase averaged images of the CH* chemiluminescence. The dynamics agreed with the common behavior described in the literature, namely frequency scaling according to Strouhal number based on flow convective timescales. However, slightly different timescales were observed for the response magnitude and phase, indicating the possibility of different scaling mechanisms at work between these phenomena. The flame heat release rate dynamics were found to be identical to the dynamic response of the flame volume to inlet velocity perturbations, suggesting a simple proportionality between heat release rate and the flame volume. This result supports the use of ensemble averaging for modeling of the turbulent velocity for predictions of flame dynamics.

1 Introduction & Background

In modern gas turbine designs, premixed combustion strategies have been employed in order to take advantage of their lower flame temperatures, and thus their reduced emissions characteristics. However, these premixed combustion strategies have been observed practically to exhibit an increased incidence of thermoacoustic combustion instabilities. Thermoacoustic instabilities are described as a closed-loop coupling between the flame heat release rate and the system acoustics. These instabilities result in high amplitude pressure and velocity oscillations which prove detrimental to the system reliability. Despite a substantial body of research on the topic, thermoacoustic instabilities continue to represent a serious practical problem in low-emission gas turbine combustion [1].

Thermoacoustic coupling may occur through a variety of coupling mechanisms. A system-level model describing two commonly observed mechanisms [2,3] is shown in Figure 1. The inner loop represents direct coupling between the flame and acoustics through velocity fluctuations. The flame heat release rate acts to drive acoustic pressure and velocity oscillations. The velocity oscillations directly excite the flame, resulting in an oscillating delivery of reactants to the flame. Pressure oscillations however, have been shown not to result in a significant coupling mechanism, thus they are not discussed further here [5]. The second mechanism, equivalence ratio coupling, occurs through interaction between the acoustic velocity and the fuel injection and mixing process. Due to their significant differences in bore sizes, fuel and air injection ports have a dissimilar acoustic response, resulting in time varying equivalence ratio pulses which may be advected to the flame. Both of these coupling modes have been observed experimentally in unstable combustion [6].

Due to the breadth of research on thermoacoustic instabilities, a wide variety of approaches to the problem exist. One approach which has received much attention [7-9,32] is that of modeling instabilities through analysis of the individual transfer functions making up the system model shown in Figure 1. Though this approach demands prior knowledge of how the individual transfer functions vary with the operating conditions, it allows a versatile framework with which to divide the problem for study.

Obtaining knowledge of the flame dynamics remains a significant area of research and many studies, both experimental [10-14,28-32] and theoretical [2,3,9,15-18], have been aimed at achieving greater understanding of the flame response to perturbations. The flame is known to behave as a low pass filter with respect to excitations, exhibiting a response at low frequencies which rolls off at higher frequencies. The transfer function's overall behavior is characterized by the low-frequency (or "DC") gain, the cutoff frequency, and the time delay between changes in the combustor inlet velocity and the observed change in heat release.

Bandwidths of the flame transfer function have been observed through both experiment and models to be a function of the Strouhal Number, based on convective length scales and the mean velocity for the flow incident on the flame [3,10]. Lohrmann and Buechner [10] showed that the appropriate length scale, x_c, for the Strouhal Number based nondimensionalization was the axial distance from the dump plane to the center of the flame. This results in a nondimensionalization of the frequency time scales by the mean time taken for the bulk flow to convect from the dump plane to the center of the flame, as shown mathematically in Equation 1.

$$St = f\tau_{conv} = f\,\frac{x_c}{\bar{u}} \tag{1}$$

The phase of flame transfer functions has been observed to exhibit a linear decline with respect to frequency, indicating time delay [10,11,14]. As with the bandwidth, the delay evident in the phase characteristic has been shown to be nondimensionalized by the same Strouhal Number as that used for the magnitude [10]. This suggests a relationship between the mechanisms causing the magnitude rolloff and the phase delay, in this case, convection of disturbances through the flame along with the mean flow. This mechanism has been observed to be closely related to the occurrence of instabilities as well [1].

The latest efforts at modeling flame dynamics have been focused on use of flame sheet models. Flame sheet models consider the flame to be a laminar flame sheet with uniform thickness which responds to the local turbulent flow field. A variety of modeling efforts have approached flame dynamics in this manner [3,15-19]. Flame sheet modeling relies on the so-called G-equation:

$$\frac{\partial G}{\partial t} = S_L\left|\nabla G\right| - \vec{u} \bullet \nabla G \tag{2}$$

This equation describes the position of the flame surface, G, as it responds to a fluctuating velocity, $\vec{u}$. Coupling between the heat release rate and the velocity occurs through wrinkling of the flame surface, which results in local increases of the flame surface area, and thus increases in the flame heat release rate. While substantial work has been done on these types of models relative to laminar flame dynamics [15,16] , models of the dynamics of the turbulence interactions with the flame have received less attention [3]. Recent efforts have attempted to model the effects of turbulence on the flame sheet using some measure of the flame surface area per unit volume [20,21], a measure which seeks to model the turbulence through decoupling of the coherent and non-coherent fluctuations. That is, the unsteady velocity is considered to be separable into a portion due to the forced oscillations and a portion due to the turbulence [3]. Thus, with phase averaging relative to the forced oscillations, the flame surface will fluctuate about a mean position, resulting in a certain volume spanned by the fluctuating surface. This is caused by the integration of the effects of the non-periodic turbulence. This volume is sometimes termed a flame brush [22]. This modeling approach has yielded results for the flame dynamics which have agreed with experimental measurements [3].

The goal of the present study was to make experimental investigations of the flame dynamics of a turbulent premixed flame relative to this paradigm, similarly to that performed by Balachandran et. al. [23]. To this end, measurements were made of the response of a turbulent, premixed flame to acoustic forcing [10-12,14]. The present study is novel in considering spatially resolved analysis of the dynamics, including measurement of the dynamic response of the flame size and spatial distribution. This analysis allowed insight into the dynamic nature of the decoupling of the turbulence (as described previously) and the practicality of its use in dynamic modeling efforts.

2 Experimental Configuration

The flame dynamics were measured using a sine dwell excitation technique in the Virginia Active Combustion Control Group (VACCG) swirl-stabilized, premixed turbulent combustor. An image of the experimental apparatus used for this study is in Figure 2, along with a dimensioned drawing of the same in Figure 3.

The combustion section used was a short quartz tube open to the atmosphere. This configuration prevented instabilities from occurring. Velocity excitations were introduced using an acoustic driver mounted in an upstream branch (as visible in both Figure 2 and Figure 3). Equivalence ratio oscillations were prevented since fuel-air mixing occurred in a line several feet upstream of injection into the base of the rig. Any equivalence ratio oscillations which did arise at the mixing point were subsequently given time to disperse prior to reaching the rig (much less the flame).

The velocity excitation was introduced using a sine dwell methodology in order to maximize the measured coherence. A single frequency was introduced at a time, over the range of 10-400 Hz. The individual frequency measurements were then combined to obtain the total flame transfer function. The amplitude of the excitation was varied as a function of frequency to ensure coherence between the input and the response. Balachandran et. al. reported [23] that the response remains linear so long as the velocity excitation remains below 15% of the mean flow. Each excitation amplitude used in this study was kept within this range. Additionally, spot test measurements were made with varying excitation amplitude to ensure that the 15% linearity criterion remained valid under the conditions and geometry used here.

The induced velocity oscillations were measured via a hotwire anemometer inserted into the flow directly upstream of the swirler, as shown in Figure 4. The hotwire was mounted on a rail to allow for repeatable insertion position in the flow. To ensure that this measurement accurately reflected the velocity incident on the flame, a transfer function was measured between the hotwire positioned immediately upstream of the flame and the hotwire positioned upstream of the swirler, which was applied to the measured transfer functions as a correction, shown in Equation 3. The hotwire correction transfer function (u'_m/u'_f) was measured from several repeated trials, but was ultimately observed to have a small effect relative to the overall dynamics. In the final comparisons between the flame heat release rate response and the flame volume response, however, the hotwire effect cancels out of the compared response.

$$\frac{q'}{u'_f} = \frac{q'}{u'_m} \cdot \frac{u'_m}{u'_f} \tag{3}$$

The flame response was characterized in two ways. First, the global flame heat release rate was measured using a PMT angled to capture radiation from the entire flame. This PMT was filtered to capture the 308 nm OH* chemiluminescence, shown to be a good indicator of the flame heat release [24,25]. Secondly, spatially resolved images of the flame yielded measurements of the flame volume and offset (discussed subsequently), which were shown to be of great interest for characterizing the flame dynamics. The images were acquired using an ICCD camera filtered at 430 nm to collect the CH* chemiluminescence, another common indicator of flame heat release rate [25]. This alternate heat release indicator was chosen for the compatibility of its visible emission with the visible spectrum optics already in use with the camera.

When acquiring the images, phase averaging was used, with 500 images averaged to produce each image of the flame. This technique was used in order to average out any oscillations in the flame structure not coherent to the velocity excitation (e.g. turbulence). In addition, the image intensities were all normalized to span the 8-bit bitmap range, to allow a universal image processing routine to be used. The camera was triggered using a square wave signal generated simultaneously with the speaker excitation. The phase of the triggering signal was varied relative to the speaker excitation by increments of 36°, resulting in ten average images over each cycle of velocity excitation.

Since the images of the flame were by nature line-of-sight, after normalization, they were deconvoluted using a three-point-fit Abel deconvolution according to the technique described by Dasch [26]. This results in an image of the averaged effective cross section of the flame. Sample before-and-after images showing the results of this technique are presented in Figure 5. The deconvolution technique introduces a degree of noise into the image, particularly along the centerline. This noise was manually removed resulting in the central black section seen in Figure 5b. The deconvoluted images were used to calculate the volume and offset, used in the transfer function calculations.

The two parameters found to be most effective as indicators of the flame distribution were the offset and the volume. The use of offset followed work by Lohrmann and Buechner [10]. For this study, the offset was calculated as the axial distance from the corrected hotwire measurement location to the intensity weighted center of mass of the flame. A sample image which shows this parameter is shown in Figure 6a. The “flame volume” was calculated from axisymmetric revolution of the cross sectional area. The cross sectional area was determined by applying a threshold to a nine-pixel average at each given pixel as described by Sanders et. al. [27]. While this thresholding technique was designed to reduce noise in the thresholding process, it was further necessary to eliminate all regions in the flame whose contiguous size was less than 500 pixels. This secondary threshold was chosen such that satellite regions of intensity (caused by stray reflections and noise) were eliminated. A sample image with the thresholding and noise reduction applied may be found in Figure 6b. To calculate the reaction region volume, this image was revolved about the centerline using shell integration. Both the volume and offset were converted to physical units using a separate calibration image of a ruler inserted into the combustion chamber.

In addition to these two primary measures of the flame distribution, an additional pair of measures were obtained, for the purpose of describing the steady shape of the flame. The length and width of the flame were measured by fitting ellipses to the left and right lobes (see Figure 6b) of the flame via the built-in "regionprops" tool from the MATLAB image processing toolbox. The length and width of the flame were considered to be the major and minor axes (respectively) of these ellipses. As stated, the length and width were used only for static characterization of the flame distribution.

All data acquisition was performed using a single computer operating on National Instruments LabView software. For the case of non-image sampled data (i.e. the hotwire and PMT signals), 30 seconds of data were acquired at each frequency with a sample rate of 7500 Hz. Anti-aliasing filters were used with a cutoff frequency of 2500 Hz. The images corresponding to this data, as mentioned, were acquired using phase locking to the sine dwell excitation signal, with 500 images averaged at each of ten phases per cycle. The rig was allowed to initially warm up for 30 minutes at each operating condition, along with 15 seconds of stabilization time following each change in excitation frequency, prior to any acquisition. Fuel and air flow rates were controlled manually and were monitored using hot film mass flow meters.

3 Data and Discussion

3.1 Steady Flame Characteristics

The first flame metrics that were considered were the steady size and shape of the reaction region. The behavior of the flame volume was considered as a function of both the mean mass flow rate and the equivalence ratio, with the results depicted in Figure 7. The volume of the reaction region was observed to increase with decreasing equivalence ratio, and to increase with increasing mass flow rate. This trend agrees with the intuitive notion that the flame volume ought to be a function of the total reaction time. Increasing mass flow rates (i.e. increasing mean velocity) would be expected to stretch the flame according to simple convection distance. Decreasing equivalence ratio corresponds to decreases in flame speed, which increase the time taken in reaching reaction completion. Note that for the highest flow rates, some deviation from expected behavior is seen at lean equivalence ratios. This is due to the flame extending beyond the end of the combustion section, limiting the reliability of these few test points.

Some insight into how the flame volume changes may be obtained by comparing changes in the flame length and width with the volume, as seen in Figure 8. From this data, it is evident that changes in the steady flame volume are proportional to the flame length. Or, that the changes in the flame volume are primarily related to elongation or contraction of the flame at a nearly constant width. Due to the shape of the flame and the combustor, these changes are primarily axial in nature. The length and width of the flame can each be described by a single curve for variations in both equivalence ratio and total mass flow rate.

Another interesting comparison is the relationship between the flame offset and the volume of the reaction region. A plot of this relationship is shown in Figure 9. As is evident, until the leanest cases where the flame extends out of the quartz section, this relationship is almost perfectly linear. It is again interesting to note that this proportionality is nearly constant even as the mean mass flow rate is varied. Given that the flame volume exists in proportion to its length as shown in Figure 8, this relationship agrees with a result stated by Lohrmann and Buechner [10], namely that the offset increases in direct proportion to the flame length.

3.2 Flame Heat Release Rate Dynamics

The frequency of the flame heat release rate dynamics was nondimensionalized according to the method described by Lohrmann and Buechner [10], using the definition of Strouhal Number found in Equation 1. Lohrmann and Buechner suggested use of the offset between the dump plane and the center of flame heat release rate as the characteristic length. Nondimensionalization of the flame dynamics measured in this study using the dump plane as the offset zero-line are shown in Figure 10a. This nondimensionalization results in good scaling for the phase at low frequency, however the magnitude nondimensionalization exhibits poor frequency scaling for several cases. It was found that by using the corrected hotwire measurement location (see Figure 4) as the zero for the offset measurement, a successful nondimensionalization for the magnitude occurred, as shown in Figure 10b. The phase is also effectively scaled, though the scaling results in a looser collapse for the data than for the in Figure 10a. This suggests that two slightly different scaling effects are at work for the magnitude and the phase. Since the dominant characteristics of both the magnitude and the phase are both scaled reasonably well using the corrected hotwire measurement location as the offset basis, this offset basis was used for the rest of the Strouhal numbers reported in this paper, according to Equation 4.

$$St_o = f\,\frac{L_o}{\bar{u}} \tag{4}$$

The linear phase characteristic is indicative of time delay between the input and the response. The Strouhal number scaling of the phase implies that this time delay is related to convection with the mean flow between the zero location and the center of flame heat release rate. Figure 11 displays the time delay calculated from the flame heat release rate transfer function as it is related to the steady flame offset for a single mass flow rate. According to the convective model, this curve should have a slope equivalent to the mean convective velocity experienced by disturbances traveling axially over the offset distance. The mean convective velocity indicated by the best fit for the data was 6.31 m/s. Due to the lack of a velocity field measurement, it is not possible to make an inference here about instantaneous convective speed relative to the mean flow.

3.3 Flame Volume dynamics

It is convenient to first consider the dynamic flame volume response on a qualitative basis. Figure 12 presents a set of images collected during excitation of the flame at 220 Hz. It is evident from the motion of the highlighted highest intensity regions that the flame is subject to forcing which moves axially through the flame. This agrees qualitatively with the mechanism found in literature, namely that flame dynamics are related to interaction of the flame with convective structures.

Figure 13 displays the flame volume’s transient response at three different excitation frequencies. This provides a glance at the data processing procedure which led to the computation of the flame transfer function. By fitting a sinusoidal wave to this data, the phase of the volume (or offset distance) response was obtained relative to the phase of the speaker excitation. Referencing the images to the speaker excitation allowed the phase of the average volume response to be determined relative to the velocity and PMT signals, which were measured simultaneously with the speaker excitation. Combining the relative phase of the volume response with the velocity normalized amplitude of the volume response yielded the flame volume transfer function.

The results of the complete flame transfer function analysis are shown for two sample data sets in Figure 14 and Figure 15. These figures compare the use of three indicators (volume, offset and intensity) from the images with the “classic” global heat release rate (HRR) response. The transfer function magnitudes are normalized to the low frequency value to allow comparison between the different scales of the measured quantities. The “intensity” measure obtained from the images was simply the summation of all intensity values in the image prior to deconvolution and was used as a direct analogy to the global HRR measurement. Since the global image intensity response matches that of the HRR dynamics, the comparison is successful. More significantly, both the volume and offset exhibit the same dynamics as the flame HRR. This means that there is a simple constant relating the volume or offset position of the flame to HRR and that no dynamic phenomena separate these characteristics.

These two data sets provide a convenient method to view similarity in the scaled flame transfer function behavior, but are unwieldy for comparison of many test conditions. Thus, to generalize the agreement between the volume and HRR dynamics, the difference between the two transfer functions was compared over broad operating conditions, as shown in Figure 16. To display each of the signals with the same frequency basis, Strouhal number is used, as in Equation 4. Perfect agreement for this comparison would be a magnitude of 0dB and 0° of phase. As is evident, reasonably good agreement exists at least up to the cutoff frequency for all test conditions. The deviations seen (particularly in phase) at higher frequencies are the result of falling signal-to-noise (and thus reductions in the coherence) at those frequencies.

This general agreement leads to a useful result for modeling the effect of turbulence on the flame heat release rate response of a flame sheet model. Using the classic realization of a flame sheet, we may write the flame heat release rate in the following manner [3,20]:

$$q = \rho \Delta H S_L A \qquad (5)$$

This may also be written considering instantaneous flame surface area (A) as a flame surface density (Σ) [20], as follows:

$$q = \int \rho \Delta H S_L \Sigma dV \qquad (6)$$

Considering the results observed in the present study (i.e. flame heat release rate dynamics were identical to the dynamics of the flame volume), the flame surface density in Equation 6 must therefore be constant in time. This implies that the turbulent oscillations in the flow field (eliminated via ensemble averaging of the velocity by You et. al. [3]) may be accounted for using this constant surface area density.

4 Summary and Conclusions

The steady characteristics of the flame size and shape were measured using averaged, deconvoluted images of the flame. The behavior of the flame obeys intuition, with leaner equivalence ratios leading to larger flames. A linear relationship was observed between the flame volume and the flame offset from the corrected hotwire measurement location. The flame width was observed to be relatively constant, with flame length being the primary contributor to changes in flame volume. The ability to measure the size characteristics of the flame for the leanest cases was limited by extension of the flame outside the combustion section.

Though the dynamics of the flame heat release rate transfer function were consistent with behavior described in literature, two slightly different frequency scaling bases were observed between the scaling for the magnitude and phase in the heat release rate response. This suggests a possibility of different scaling mechanisms at work for each of these components of the response. Further investigation is necessary to differentiate between these effects. Time delay in the phase was found to be linearly related to the offset from the corrected hotwire measurement location. This proportionality scaled with the average upstream axial velocity, further implying a link between the dynamics and convected structures in the flow.

The dynamics of the flame volume were measured at a variety of operating conditions using phase averaged images. These volume dynamics were found to be virtually identical to the flame heat release rate dynamics. This relationship is extremely important to use of turbulence ensemble averaging in flame dynamic modeling efforts, since it implies that the flame surface density may be constant with respect to time in the presence of acoustic forcing. The constant relationship between the flame volume and flame heat release rate dynamics thus suggests that ensemble averaging does not introduce or neglect any dynamics salient to those of the heat release rate response. Therefore ensemble averaging of the turbulence should be a suitable method of velocity modeling for prediction of flame dynamics. This work should be extended to consider the response of the flame to equivalence ratio oscillations.

Nomenclature

Φ	equivalence ratio	[-]
q	heat release rate	[kW]
u	flow velocity	[m/s]
St	Strouhal number	[-]
f	frequency	[Hz]
τ_{conv}	characteristic convective time	[s]
x_c	general characteristic length	[m]
S_L	laminar flame speed	[m/s]
G	flame surface scalar	[-]
L_o	offset to center of flame heat release rate	[m]
ρ	density	[kg/m^3]
ΔH	enthalpy release per unit mass of fuel	[kJ/kg]
A	flame surface area	[m^2]
Σ	flame surface density	[m^2/m^3]
V	flame volume	[m^3]
Sub and Superscripts		
x_f	variable x at the flame location	
x_m	variable x at the hotwire measurement location	

x’	oscillations in variable x
$\bar{x}$	mean component of variable x

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